IGCSE Higher
Sheet H4-1  2-07b-1 Completing the Square - Answers

1.
(a)  \((x + 4)^2 - 9\)
(b)  \((x + 6)^2 - 11\)
(c)  \((x + 9)^2 - 6\)
(d)  \((x + 3)^2 - 4\)
(e)  \((x + 5)^2 - 18\)
(f)  \((x + 6)^2 - 33\)
(g)  \((x + 1)^2 - 2\)
(h)  \((x + 4)^2 - 19\)
(i)  \((x + 2)^2 - 3\)
(j)  \((x + 7)^2 - 9\)

2.
(a)  \((x - 4)^2 - 5\)
(b)  \((x - 6)^2 - 23\)
(c)  \((x - 9)^2 - 21\)
(d)  \((x + 3)^2 - 3\)
(e)  \((x - 5)^2 - 40\)
(f)  \((x + 6)^2 - 7\)
(g)  \((x - 1)^2 + 2\)
(h)  \((x + 4)^2 - 3\)
(i)  \((x - 2)^2 - 14\)
(j)  \((x + 7)^2 - 52\)

3.
(a)  \(x = -1 \pm \sqrt{2}\)
(b)  \(x = 2 \pm \sqrt{7}\)
(c)  \(x = -6\)
(d)  \(x = -10 \pm \sqrt{95}\)
(e)  \(x = 1, x = -9\)
(f)  \(x = 1 \pm \sqrt{8}\)

4.
(a)  \(x = -3, x = 1\)
(b)  \(x = -3, x = 1\)
(c)  \(x = -5 \pm \sqrt{23}/2\)
(d)  \(x = -2 \pm \sqrt{10}\)

5.
(a)  \(-4\)
(b)  \(-22\)
(c)  \(-3\)

6.
(a)  \((-3, -10)\)
(b)  \((1, 4)\)
(c)  \((-3, 0)\)
(d)  \(\left(-\frac{1}{2}, -\frac{5}{4}\right)\)
1. 
(a) \( x = -2 \pm \sqrt{3} \)  
(b) \( x = 3 \pm \sqrt{2} \)  
(c) \( x = 5 \pm \sqrt{5} \)  
(d) \( x = -4 \pm \sqrt{5} \)  
(e) \( h = -1 \pm \sqrt{8} \)  
(f) \( e = -5 \)

2. 
(a) \( x = \frac{-3 \pm \sqrt{13}}{2} \)  
(b) \( x = \frac{-5 \pm \sqrt{57}}{2} \)  
(c) \( x = \frac{-7 \pm \sqrt{37}}{2} \)  
(d) \( x = \frac{-1 \pm \sqrt{3}}{2} \)  
(e) \( x = \frac{-11 \pm \sqrt{117}}{2} \)  
(f) \( x = \frac{-7 \pm \sqrt{61}}{2} \)  
(g) \( x = -1 \pm \sqrt{2} \)  
(f) \( x = 2 \pm \sqrt{7} \)

3. \( x = \frac{-1 \pm \sqrt{41}}{4} \)

4. \( x = \frac{-7 \pm \sqrt{37}}{6} \)

5. 
(a) \( \left( \frac{1}{4}, \frac{9}{8} \right) \)  
(b) \( \left( \frac{-7}{6}, \frac{-109}{12} \right) \)

6. 
(a) \( y = 3 \)  
(b) None  
(c) 2

7. 
(a) \( y = x^2 + x - 2 \)  
(b) \( y = 6 + x - x^3 \)  
(c) \( y = 2x^2 - 8x - 10 \)  
(d) \( y = -3x^2 - 9x + 30 \)
Sheet H4-3  2-07b-3 Quadratic Formula - Answers

1. (a) 16  (b) 0  
   (c) 97  (d) 25  
   (e) 149  (f) 125  

2.  
   (a) \(-\frac{6 \pm \sqrt{16}}{2}\)  
   (b) \(\frac{8 \pm \sqrt{0}}{2}\)  
   (c) \(-\frac{3 \pm \sqrt{97}}{4}\)  
   (d) \(-\frac{7 \pm \sqrt{25}}{4}\)  
   (e) \(\frac{3 \pm \sqrt{49}}{10}\)  
   (f) \(\frac{11 \pm \sqrt{25}}{2}\)  

3. (a) -1 or -5  
   (b) 4  
   (c) 1.71 or -3.21  
   (d) -0.5 or -3  
   (e) 1.52 or -0.92  
   (f) 11.1 or -0.0902  

4.  
   (a) \(-\frac{5 \pm \sqrt{21}}{2}\)  
   (b) \(-\frac{3 \pm \sqrt{17}}{2}\)  
   (c) \(\frac{3 \pm \sqrt{29}}{10}\)  
   (d) \(-\frac{1 \pm \sqrt{5}}{4}\)  
   (e) \(-\frac{5 \pm \sqrt{221}}{14}\)  
   (f) \(-\frac{8 \pm \sqrt{108}}{2}\)  

5.  
   (a) \(x = \frac{-5 \pm \sqrt{13}}{6}\) = -1.43 (to 3sf) or -0.232 (to 3sf)  
   (b) \(z = \frac{-10 \pm \sqrt{128}}{14}\) = -1.52 (to 3sf) or 0.0938 (to 3sf)  
   (c) \(c = \frac{2 \pm \sqrt{64}}{10}\) = -0.717 (to 3sf) or 1.12 (to 3sf)  
   (d) \(q = \frac{-1 \pm \sqrt{33}}{4}\) = -1.69 (to 3sf) or 1.19 (to 3sf)  
   (e) \(u = \frac{-9 \pm \sqrt{33}}{8}\) = -1.84 (to 3sf) or -0.407 (to 3sf)  
   (f) \(t = \frac{-20 \pm \sqrt{40}}{26}\) = -1.22 (to 3sf) or -0.314 (to 3sf)  

(cont.)

6.  
   (a) \(x = \frac{11 \pm \sqrt{125}}{2}\) = -0.0902 (to 3sf) or 11.1 (to 3sf)  
   (b) \(x = \frac{-17 \pm \sqrt{1349}}{10}\) = -5.37 (to 3sf) or 1.97 (to 3sf)  
   (c) \(x = \frac{-19 \pm \sqrt{537}}{8}\) = -5.27 (to 3sf) or 0.522 (to 3sf)  
   (d) \(x = \frac{5 \pm \sqrt{389}}{14}\) = -1.05 (to 3sf) or 1.77 (to 3sf)  

7.  
   (a) \(x = \frac{-11 \pm \sqrt{17}}{2}\) = -10.9 (to 3sf) or -0.917 (to 3sf)  
   (b) \(x = \frac{-9 \pm \sqrt{105}}{4}\) = -4.81 (to 3sf) or 0.311 (to 3sf)  
   (c) \(x = \frac{-7 \pm \sqrt{157}}{6}\) = -3.26 (to 3sf) or 0.922 (to 3sf)  
   (d) \(x = \frac{3 \pm \sqrt{24}}{5}\) = 0.380 (to 3sf) or 1.58 (to 3sf)  
   (e) \(x = \frac{-1 \pm \sqrt{17}}{4}\) = -1.28 (to 3sf) or 0.781 (to 3sf)  
   (f) \(x = \frac{-13 \pm \sqrt{217}}{12}\) = -2.31 (to 3sf) or 0.144 (to 3sf)  

8.  
   (a) \(x = -1.71\) (to 3sf) or 3.21 (to 3sf)  
   (b) \(x = -4.04\) (to 3sf) or 1.64 (to 3sf)  
   (c) \(x = -4.64\) (to 3sf) or 3.07 (to 3sf)  
   (d) \(x = -0.0744\) (to 3sf) or 2.24 (to 3sf)
Sheet H4-4 2-07b-4 Quadratic Formula - Answers

1. (a) \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -0.807 \) (to 3sf) or \(-6.19\) (to 3sf) 
(b) \( y = \frac{-2 \pm \sqrt{8}}{2} = 0.414 \) (to 3sf) or \(-2.41\) (to 3sf) 
(c) \( a = \frac{3 \pm \sqrt{37}}{2} = 4.54 \) (to 3sf) or \(-1.54\) (to 3sf) 
(d) \( h = \frac{-1 \pm \sqrt{41}}{10} = 0.540 \) (to 3sf) or \(-0.740\) (to 3sf) 
(e) \( d = \frac{3 \pm \sqrt{185}}{8} = 2.08 \) (to 3sf) or \(-1.33\) (to 3sf) 
(f) \( z = \frac{7 \pm \sqrt{89}}{4} = 0.608 \) (to 3sf) or \(-4.11\) (to 3sf)

2. (a) \( p = \frac{-7 \pm \sqrt{337}}{18} = 0.631 \) (to 3sf) or \(-1.41\) (to 3sf) 
(b) \( k = \frac{3 \pm \sqrt{385}}{8} = 2.08 \) (to 3sf) or \(-1.33\) (to 3sf) 
(c) \( z = \frac{-7 \pm \sqrt{109}}{6} = 0.573 \) (to 3sf) or \(-2.91\) (to 3sf) 
(d) \( y = \frac{-3 \pm \sqrt{57}}{12} = 0.379 \) (to 3sf) or \(-0.879\) (to 3sf) 
(e) \( t = \frac{7 \pm \sqrt{489}}{22} = 1.32 \) (to 3sf) or \(-0.687\) (to 3sf) 
(f) \( u = \frac{-1 \pm \sqrt{141}}{14} = 0.777 \) (to 3sf) or \(-0.920\) (to 3sf) 
(g) \( q = \frac{-4 \pm \sqrt{32}}{2} = 0.828 \) (to 3sf) or \(-4.83\) (to 3sf) 
(h) \( x = \frac{-10 \pm \sqrt{136}}{6} = 0.277 \) (to 3sf) or \(-3.61\) (to 3sf) 
(i) \( x = \frac{-7 \pm \sqrt{113}}{16} = 0.227 \) (to 3sf) or \(-1.10\) (to 3sf) 
(j) \( x = \frac{5 \pm \sqrt{20}}{2} = 5.37 \) (to 3sf) or \(-0.372\) (to 3sf) 
(k) \( x = 2 \pm \sqrt{27} \approx -3.20 \) (to 3sf) or \(7.20\) (to 3sf) 
(l) \( x = \frac{16 \pm \sqrt{48}}{4} = 2.27 \) (to 3sf) or \(5.73\) (to 3sf)

3. \( c < 16 \)
4. \( \pm 6 \)
5. \( a > 16 \)
1. (a) \( a = -2, a = -5 \)
(b) \( w = -2, w = -8 \)
(c) \( x = 3, x = 5 \)
(d) \( y = 1 \)
(e) \( h = 2, h = -7 \)
(f) \( r = 2, r = -5 \)
(g) \( t = -2, t = 12 \)
(h) \( k = 2, k = -9 \)
(i) \( y = -4, y = 4 \)
(j) \( m = 0, m = -2 \)
(k) \( u = 0, u = 5 \)
(l) \( k = -6, k = 6 \)

2. (a) \( a = 5, a = -7 \)
(b) \( c = 4, c = -7 \)

3. (a) \( x + 3 \)
(b) \( - \)
(c) \( x = 5, x = -8 \)
(d) \( 5 \text{ cm} \)

4. (a) \( w = -5 \)
(b) \( w^2 - 5w - 24 = 0 \)
(c) \( w = -3, w = 8 \)
(d) \( 8 \text{ cm} \)

5. (a) \( h + 4 \)
(b) \( h^2 + 4h - 60 = 0 \)
(c) \( h = -10, h = 6 \)
(d) \( 6 \text{ cm} \)

6. (a) \( x^2 + 3x = 6x \)
(b) \( x = 0, x = 3 \)
(c) \( 3 \)

7. (a) \( x = -1, x = -2 \)
(b) \( x = 3, x = 4 \)
Sheet H4-7  2-07c-03 Quadratic Factorisation-Problems - Answers

1. 
   (a) \( 14 - w \)  
   (b) \( w^2 - 14w + 40 = 0 \)  
   (c) 4m by 10m

2. 
   (a) \( 12 - 2x \) by \( 8 - 2x \)  
   (b) \( (12 - 2x)(8 - 2x) \)  
   (c) \( 4x^2 - 40x + 64 = 0 \)  
   (d) \( - \)  
   (e) \( x = 2 \) (\( x \) cannot be greater than 4)

3. 
   (a) \( x + 7 \)  
   (b) \( 2x^2 + 14x - 120 = 0 \)  
   (c) \( x = 5 \)

4. 
   (a) \( x + 3 \)  
   (b) \( \frac{x(x + 3)}{2} \)  
   (c) \( x^2 + 3x - 54 = 0 \)  
   (d) \( x - 6 \)

5. \( x = 4 \)

6. 
   (a) 8.75  
   (b) \( x = \frac{1}{2} \)

7. \( x = 3 \)

Sheet H4-8  2-07c-04 Quadratic Factorisation-Problems - Answers

1. 
   (a) \( 25 - w \)  
   (b) \( w(25 - w) = 154 \)  
   (c) 11m or 14m

2. 
   (a) \( 2x - 3 \)  
   (b) \( x(2x - 3) = 35 \)  
   (c) \( x = 5 \)

3. 
   (a) \( 2h + 3 \)  
   (b) \( \frac{h(2h + 3)}{2} = 76 \)  
   (c) 8cm

4. 
   (a) \( x + 7 \)  
   (b) \( x^2 + (x + 7)^2 = 169 \)  
   (c) \( x = 5 \)

5. 
   (a) \( (s - 3)(s - 4) = 20 \)  
   (b) \( s = 8 \)
Sheet H4-9  2-07c-05 Quadratic Factorisation-Problems - Answers

1. 15m
2. 10cm
3. 8 and 8.5
4. 4
5. 3
6. 18
7. 21cm
8. 15m by 20m
9. 7
10. 3cm by 4cm by 12cm

Sheet H4-10  2-07c-06 Quadratic Formula-Problems - Answers

1. (a) \( x + 2 \)
   (b) \( x = 1.45 \)

2. (a) \( (2x + 3)^2 = 4x^2 + x^2 \)
   (b) -
   (c) 0.517

3. (a) \( x(x + 50) = 56496 \)
   (b) \( x = 214 \) m and so 956m of fencing

4. 4.09m by 7.09m
5. 2.87cm
6. 46.3m
7. 226m by 124m
8. 4.24
9. \( \frac{1 + \sqrt{5}}{2} \)
10. (a) \( 2x + 1 \)
    (b) \( x^2 + 7^2 = (2x + 1)^2 \)
    (c) 24.2
11. (a) \( x + 23 \)
    (b) \( x^2 + (x + 23)^2 = 65^2 \)
    (c) 33
12. (a) \( x^2 + (x + 70)^2 = 180^2 \)
    (b) 87.4m
13. (a) \( x^2 + (x + 17)^2 = 305^2 \)
    (b) 207
14. (a) \( x^2 + (x + 34)^2 = 50^2 \)
    (b) \( x = 14 \)
15. (a) -
    (b) 7.33cm
**Sheet H4-11  2-07c-07 Quadratic Equations Revision - Answers**

1. (a) \((x + 3)(x + 5)\)  
(b) \((x + 2)(x + 4)\)  
(c) \((x + 3)(2x + 3)\)  
(d) \((x - 4)(3x + 2)\)  
(e) \((x - 1)(5x - 4)\)  
(f) \((2x - 3)(3x + 1)\)  

2. (a) \(x = -3, x = -4\)  
(b) \(x = 7, x = -2\)  
(c) \(x = \frac{1}{3}, x = -2\)  
(d) \(x = \frac{3}{2}, x = -\frac{3}{2}\)  
(e) \(x = 0, x = \frac{7}{2}\)  
(f) \(x = -\frac{1}{8}, x = 2\)  

3. (a) \(x = \frac{-3 + \sqrt{17}}{4} = 0.281\) (to 3sf) or \(-1.78\) (to 3sf)  
(b) \(x = \frac{5 + \sqrt{37}}{2} = 5.54\) (to 3sf) or \(-0.541\) (to 3sf)  
(c) \(x = \frac{-2 + \sqrt{144}}{10} = 1\) or \(-1.4\)  
(d) \(x = \frac{-11 + \sqrt{273}}{4} = 1.38\) (to 3sf) or \(-6.88\) (to 3sf)  
(e) \(x = \frac{-9 + \sqrt{121}}{10} = 0.2\) or \(-2\)  
(f) \(x = \frac{-5 + \sqrt{181}}{2} = 4.23\) (to 3sf) or \(-9.23\) (to 3sf)  

4. (a) \(x = \frac{28}{4} = 7\)  
(b) \(x = \frac{22}{7} = 3\)  
(c) \(x = \frac{65}{m} = 6.13\)  

5. (a) \(x + 7\)  
(b) \(x^2 + (x + 7)^2 = 97^2\)  
(c) \(65m\)  

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**Sheet H4-12  2-07c-08 Quadratic Equations Revision - Answers**

1. (a) \((x + 2)(x + 4)\)  
(b) \((x + 3)(x + 5)\)  
(c) \((x + 4)(x - 3)\)  
(d) \((x - 4)(x - 5)\)  
(e) \((x + 2)(x - 2)\)  
(f) \((x + 5)(x - 5)\)  

2. (a) \((x + 2)(2x + 3)\)  
(b) \((5x - 2)(x - 2)\)  
(c) \((3x + 2)(x - 5)\)  
(d) \((4x + 3)(4x - 3)\)  

3. (a) \(x = -1, x = -4\)  
(b) \(x = 4, x = -3\)  
(c) \(x = \frac{1}{2}, x = \frac{-7}{2}\)  

4. (a) \(x = \frac{-2 \pm \sqrt{64}}{10} = 0.717\) (to 3sf) or \(-1.12\) (to 3sf)  
(b) \(x = \frac{-1 \pm \sqrt{4}}{4} = 1.35\) (to 3sf) or \(-1.85\) (to 3sf)  
(c) \(x = \frac{1 \pm \sqrt{3}}{6} = 0.768\) (to 3sf) or \(-0.434\) (to 3sf)  
(d) \(x = \frac{-2 \pm \sqrt{72}}{2} = -5.24\) (to 3sf) or \(3.24\) (to 3sf)  

5. (a) \(x + 3\)  
(b) \(-3\)  
(c) \(6.13\)
Sheet H4-13  2-07c-09 Quadratic Equations Revision - Answers

1. 
(a) \((x + 3)(x + 7)\)  (b) \((t + 3)(t + 11)\)  (c) \((y - 4)(y + 9)\)  
(d) \((r - 12)(r + 5)\)  (e) \((a - 6)(a + 8)\)  (f) \((w - 6)(w + 1)\)  
(g) \((2b + 3)(3b - 1)\)  (h) \(3(2u + 1)(2u - 1)\)  (i) \(5(3q + 1)^2\)

2. 
(a) \(x = -2, x = -9\)  (b) \(x = -1, x = 5\)  
(c) \(x = 2, x = 12\)  (d) \(x = -1\)  
(e) \(x = \frac{1}{2}, x = -1\)  (f) \(x = \frac{2}{5}, x = -2\)

3. 
(a) \(x = \frac{-11 \pm \sqrt{113}}{2}\)  
(b) \(y = \frac{-5 \pm \sqrt{21}}{2}\)  
(c) \(g = \frac{-10 \pm \sqrt{136}}{2}\)  
(d) \(t = \frac{17 \pm \sqrt{669}}{10}\)  
(e) \(r = \frac{-9 \pm \sqrt{77}}{4}\)  
(f) \(u = \frac{5 \pm \sqrt{77}}{6}\)

4. 
(a) \(-\)  
(b) \(x(110 - x) = 2500\)  
(c) \(78\)m by \(32\)m

5. 
(a) \(-\)  
(b) \(60\) km/h

6. 
(a) \(c = -1, c = -7\)  
(b) \(x = 2.62, x = 0.382\)  
(c) \(x = 0.618, x = -1.62\)  
(d) \(x = -2.94, x = 2.27\)

7. \(10.9\)cm

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Sheet H4-15  2-07d-02 Quadratic Simultaneous Equations - Answers

1. (a) \( x = -2, \ y = -5 \)  \( x = 2, \ y = 5 \)
   (b) \( x = -5, \ y = -3 \)  \( x = 3, \ y = 5 \)
   (c) \( x = -6, \ y = -2 \)  \( x = 2, \ y = 6 \)
   (d) \( x = 5, \ y = 11 \)  \( x = -7.5, \ y = -14 \)

2. (a) \( x = 3, \ y = 4 \)  \( x = 4, \ y = 3 \)
   (b) \( x = 2, \ y = 8 \)  \( x = -\frac{7}{2}, \ y = -\frac{17}{2} \)
   (c) \( x = 10, \ y = 2 \)  \( x = -2, \ y = -10 \)
   (d) \( x = 5, \ y = 4 \)  \( x = 4, \ y = 5 \)

3. \( x = -1, \ y = 0 \)  \( x = 7, \ y = 2 \)

4. \( x = -\frac{8}{3}, \ y = -\frac{1}{3} \)  \( x = 4, \ y = 3 \)

Sheet H4-16  2-08a-1 Quadratic Inequalities - Answers

1. (a) \( w \leq 2 \)  \( x < -4 \)
   (b) \( 2 < y \leq 5 \)  \( -9 < z < -3 \)
   (c) \( -1 < x < 1 \)  \( 2 < y < 5 \)
   (d) \( -3 \leq x \leq 3 \)  \( -4 < x < 4 \)

2. (a) \( -1 < x < 1 \)  \( x < -4, \ x > 4 \)
   (b) \( -3 \leq x \leq 3 \)  \( 2 < y < 5 \)
   (c) \( x \leq -5, \ x > 5 \)  \( x \leq -4, \ x \geq 4 \)

3. (a) \( x \leq -2, \ x \geq 6 \)  \( -4 < x < -2 \)
   (b) \( x \leq -1, \ x \geq 6 \)  \( -3 < x < -2 \)
   (c) \( x \leq 3, \ x \geq 4 \)  \( 1 \leq x \leq 2 \)

4. (a) \( 1.5 < x < 4 \)  \( \frac{2}{3} < x < 5 \)
   (b) \( \frac{9}{2} < x < \frac{1}{2} \)  \( \frac{4}{3} \leq x \geq 3 \)

5. (a) \( x \leq -\frac{1}{2}, \ x \geq \frac{3}{2} \)  \( -2 < x < \frac{13}{3} \)
   (b) \( 2 < x < \frac{5}{2} \)  \( -2 < x < \frac{13}{3} \)
   (c) \( 2 < x < \frac{5}{2} \)  \( \frac{5}{2} \leq x \geq 3 \)
Sheet H4-17   2-08b-1 Graphical Inequalities - Answers

1. (a) Line A \( y = x + 4 \)
   Line B \( y = 9 - 2x \)
   Line C \( y = 4 \)

(b) 

(c) \( y \leq 3x + 4 \) \( y \geq 4 \) \( y \leq 9 - 2x \)

2. 

(b) 

(c) \((0, 6), (1, 2), (8, 14), (0, 10)\)

Sheet H4-18   2-08b-2 Graphical Inequalities - Answers

1. (a) 

2. 

(a) 

(b) 

(c) 

\( y = x + 1 \)
\( y = -x - 7 \)
\( y = 4x - 12 \)

(b) 

\( y \leq x + 1 \) \( y \geq -x - 7 \) \( y \geq 4x - 12 \)
1. Graph of $3x + y \leq 12$

2. Graph of $y \geq x - 2$

3. Graph of $y \leq 2x + 6$

4. Graph of $y + 16x \geq 16$

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1. (a)  

(b) (-1, 3), (4, 3), (4, 4), (2, 6)  

(c) (-1, 2), (4, 3), (4, 4), (2, 6)

2. (a) 

(b) 

(c) (-1, 3), (4, 3), (4, 4), (2, 6)

3. (a) Line A \( y = 16 - 2x \)  
Line B \( y = 10 - 2x \)  
Line C \( y = x + 5 \)  
Line D \( y = x - 5 \)

(b) 

(c) Parallelogram

4. (a) \( 3x + 4y \leq 48 \) \( 4x + 3y \leq 60 \) \( x \geq 0 \) \( y \geq 0 \)

(b) -